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APPLICATION OF THE POWER-LIMITED PROBLEM TO THE ELECTRIC PROPULSION MISSION DESIGN

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Abstract

Two mathematical propulsion models have most frequently been used[1]. In the constant ejection velocity (CEV) model, either the thrust or thrust acceleration is bounded. In the power-limited (LP) model, power of the jet (the product of the thrust magnitude and the specific impulse) is bounded. This paper is concerned to comparison of the LP and CEV models. It is demonstrated that LP solution is a quite precise approximation of the CEV one. The possibility to use LP solution to solve CEV problem is considered. The corresponding continuation method is presented.

Nomenclature

CEV = constant ejection velocity model
LP = power limited model
TPBVP = two point boundary value problem
$\mathbf{x}$ = vector of the spacecraft's position
$\mathbf{v}$ = vector of the spacecraft's velocity
$\mathbf{a}$ = vector of the thrust acceleration
a = magnitude of the thrust acceleration
ε = switching function
$\mathbf{p}_v$ = adjoint vector
t = current time
t_o = launch date
T = transfer duration
T_b = total duration of the burn arcs
Ω = force function of the gravity field
J = performance index
V_∞ = initial asymptotic geocentric velocity of the spacecraft
m_o = initial mass of the spacecraft at the beginning of the heliocentric arc
m_k = final mass of the spacecraft
Δ = relatively difference between LP and CEV final masses
$\mathbf{z}$ = vector of the unknown TPBVP parameters (adjoint vector and its derivative)
$\mathbf{f}$ = vector of the right boundary conditions
$\mathbf{b}$ = vector of the initial residuals
φ_i = continuation functions
τ = continuation parameter
k = normalizing parameter

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Introduction

Two conventional mathematical models of the low-thrust transfer are discussed. The first one assumes that thrust magnitude is controlled and power limit is unique constraint (LP model). Problem of the mission optimization is separated into two subproblems within this model -- dynamical and parametric ones. The second mathematical model assumes that thrust or thrust acceleration magnitude is constant. It is CEV model. Problem is not separated in this case of optimization, and process of the mission design becomes an iterative one. The first approach (LP problem) is essentially more regular than the second one (CEV problem). Unfortunately, possibility of the thrust control often very limited in the practical applications. So, the second approach is more realistic.

Selection of the main design parameters within the collection of all mission parameters allows to improve design process. This selection based on compressed information that concerns main performances of the spacecraft. The main approach is consists in separation of the complex system into base and detailed systems. Structure of the mission model becomes hierarchical one within this approach. Main design parameters coordinate design process. Detailed optimization of the mission is carried out in assumption that these parameters are fixed. All main spacecraft subsystems are presented in the simplified form as so-called design relations. The design relations are functions of the main design parameters and some statistical coefficients. Spacecraft model in combination with dynamical one presents the mission model. Concordance of the main design parameters with trajectory and control parameters is the main problem of the mission design. The main mathematical difficulty of the low-thrust mission design consists in the trajectory optimization. Optimal control problem should be solved every time when it varies set of the main design parameters.

Main advantage of the LP problem consists in the possibility of it separation. If optimal trajectory is found, mass optimization does not require recalculation of this trajectory. Several ways exist to use this advantage for CEV mission optimization.

Comparison of the LP- and CEV-model. Use of LP solution to estimate solution of the CEV problem

The easiest way is to use LP model for estimation main parameters of the CEV mission. It is necessary to estimate main design parameters -- payload mass, fuel consumption, thrust magnitude, ejection velocity, transfer time, etc. Analysis, which was worked out, shows possibility of this way. Authors carried out comparison of the LP planetary missions and the CEV ones. Difference between main parameters of these missions mostly remains within a few percents.

For example, let us consider Fortuna rendezvous mission (Fortuna is asteroid, which belongs to main asteroid belt). It is assumed that spacecraft delivers into the geocentric hyperbolic orbit by means of Russian launcher “Proton” and upper stage “Block D”. The magnitude of the asymptotic velocity V_∞ of this orbit and launch date t_0 is optimized. Both LP and CEV model are used to optimize heliocentric arc of trajectory. It is assumed that thrust magnitude in case of CEV problem is 0.9885 N. Some results are presented in the table 1. The last column of the table 1 contents relatively difference between final mass of spacecraft in cases of using LP and CEV model. This difference remains within a few percents.

Table 1

Fortuna rendezvous missions

N	t_o	T [d]	V_∞ [m/s]	m_o [kg]	CEV model		LP model	Δ
					T_b [d]	m_k^{CEV} [kg]	m_k^{LP} [kg]	
1	20.7.2000	900	726	6127	731	4951	5089	2.8 %
2	25.6.2000	1050	466	6160	665	5090	5262	3.4 %
3	20.6.2000	1200	519	6155	645	5117	5373	5.0 %

Fig.1 presents heliocentric arc of LP trajectory. Strokes along trajectory denote magnitude and direction of the thrust acceleration. The CEV trajectory is not distinguished from the LP one in presented scale.

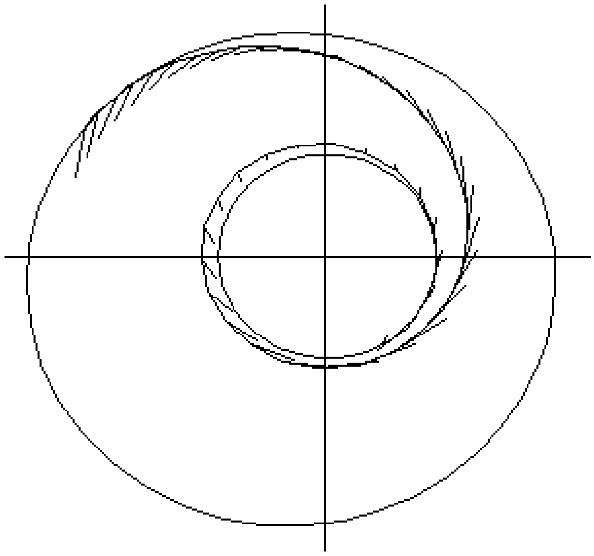

Fig. 1. Fortuna rendezvous mission (LP model, N=1 in the table 1)

Fig.2 presents thrust acceleration with respect to time in cases of LP and CEV model. It is obviously that burn arcs of the CEV model corresponds to the local maximums of the LP model.

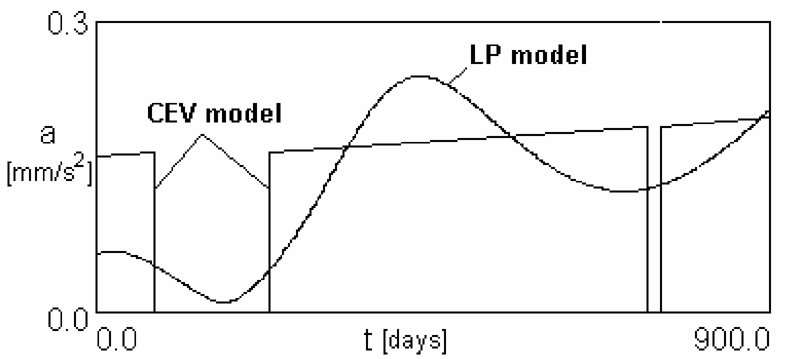

Fig. 2. Comparison of the CEV and LP thrust acceleration profiles (Fortuna rendezvous mission, N=1 in the table 1)

Continuation method to transform LP solution into CEV one

The second way of the LP-model application is continuation of the its solution into the solution of the CEV problem. It is assumed that spacecraft moves in the force field Ω. Equation of spacecraft motion in the inertial Cartesian coordinates is follows:

$$\mathrm{d}^2\mathbf{x}/\mathrm{d}t^2 = \Omega_\mathbf{x} + \varepsilon\mathbf{a}, \tag{1}$$

where $\mathbf{x}=(x, y, z)^\mathrm{T}$ - vector of the spacecraft position, $\mathbf{a}$ - vector of the thrust acceleration, ε - switching function (ε = 1 if thrusters are running, otherwise ε = 0) The minimum-fuel transfer problem is reduced to the minimization of the performance index[1]

$$J = \frac{1}{2}\int_0^T \varepsilon\mathbf{a}^\mathrm{T}\mathbf{a}\, dt \tag{2}$$

in case of LP problem and

$$J = \int_0^T \varepsilon a\, dt, \tag{3}$$

in case of CEV one. T is transfer duration and $a = |\mathbf{a}|$ here. It is assumed that transfer duration is fixed. It is assumed that thrust acceleration a = const in case of CEV problem.

Optimal control problem is reduced to the two point boundary value problem by means of Pontryagin's maximum principle. Optimal control is

$$\mathbf{a} = \mathbf{p}_\mathbf{v},\ \varepsilon \equiv 1 \tag{4}$$

in case of LP problem and it is

$$\mathbf{a} = a\mathbf{p}_\mathrm{v}/p_\mathrm{v},\ \varepsilon = \varepsilon(p_\mathrm{v} -1) = \begin{cases} 1 & \text{if } \mathbf{p}_\mathrm{v} > 1 \text{ and} \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

in case of CEV one, where $p_\mathrm{v} = |\mathbf{p}_\mathbf{v}|$.. Adjoint vector $\mathbf{p}_\mathbf{v}$ complies with the differential equation

$$\mathrm{d}^2\mathbf{p}_\mathbf{v}/\mathrm{d}t^2 = \Omega_\mathbf{xx}\,\mathbf{p}_\mathbf{v}, \tag{6}$$

Boundary and transversality conditions complete formulation of the TPBVP. For example, let us consider optimization of the heliocentric arc of the interplanetary mission. Boundary conditions at the t=0 are[3]

$$\mathbf{x}(0) = \mathbf{x}_\mathrm{o},\ \mathrm{d}\mathbf{x}(0)/\mathrm{d}t = \mathbf{v}_\mathrm{o} + V_\infty\,\mathbf{p}_\mathbf{v}/p_v, \tag{6}$$

and ones at the $t = T$ are

$$\mathbf{x}(T) = \mathbf{x}_\mathrm{k},\ \mathrm{d}\mathbf{x}(T)/\mathrm{d}t = \mathbf{v}_\mathrm{k} \tag{7}$$

in case of rendezvous mission or

$$\mathbf{x}(T) = \mathbf{x}_\mathrm{k},\ \mathbf{p}_\mathbf{v}(T) = 0 \tag{8}$$

in case of flyby mission, where V_∞ is initial asymptotic geocentric velocity of the spacecraft.

Let us make use of the basic ideas of continuation method to transform LP problem to the CEV one. Equation of the optimal motion may be written follows in the general case:

$$\left.\begin{aligned}\ddot{\mathbf{x}} &= \Omega_{\mathbf{x}} + \varphi_1(\tau)\mathbf{p}_{\mathrm{v}} + \varphi_2(\tau)\frac{a\varepsilon}{p_v}\mathbf{p}_{\mathrm{v}},\\ \ddot{\mathbf{p}}_{\mathrm{v}} &= \Omega_{\mathbf{xx}}\mathbf{p}_{\mathrm{v}},\end{aligned}\right\} \tag{9}$$

where $\varphi_1(\tau)$, $\varphi_2(\tau)$ are continuation functions, so that eq. (9) transforms into the equation of the LP optimal motion if τ=0:

$$\left.\begin{aligned}\ddot{\mathbf{x}} &= \Omega_{\mathbf{x}} + \mathbf{p}_{\mathrm{v}},\\ \ddot{\mathbf{p}}_{\mathrm{v}} &= \Omega_{\mathbf{xx}}\mathbf{p}_{\mathrm{v}},\end{aligned}\right\} \tag{10}$$

and it transforms into the equation of the CEV optimal motion if τ=1:

$$\left.\begin{aligned}\ddot{\mathbf{x}} &= \Omega_{\mathbf{x}} + \frac{a\varepsilon}{p_v}\mathbf{p}_{\mathrm{v}},\\ \ddot{\mathbf{p}}_{\mathrm{v}} &= \Omega_{\mathbf{xx}}\mathbf{p}_{\mathrm{v}},\end{aligned}\right\} \tag{11}$$

It is necessary to solve equation

$$\mathbf{f}(t,\ \tau,\ \mathbf{p}_{\mathrm{vo}},\ \dot{\mathbf{p}}_{\mathrm{vo}})|_{t=T,\ \tau=1} = 0, \tag{12}$$

with respect to unknown initial value of the adjoint vector $\mathbf{p}_{\mathrm{vo}}$ and its derivative $\dot{\mathbf{p}}_{\mathrm{vo}}$, where function $\mathbf{f}(t,\ \tau,\ \mathbf{p}_{\mathrm{vo}},\ \dot{\mathbf{p}}_{\mathrm{vo}})$ is equal to

$$\mathbf{f} = \begin{pmatrix} \mathbf{x}(T) - \mathbf{x}_{\mathrm{k}} \\ \mathrm{d}\,\mathbf{x}(T)/\mathrm{d}\,t - \mathbf{v}_{\mathrm{k}} \end{pmatrix} \tag{13}$$

in case of rendezvous mission or

$$\mathbf{f} = \begin{pmatrix} \mathbf{x}(T) - \mathbf{x}_{\mathrm{k}} \\ \mathbf{p}_{\mathrm{v}}(T) \end{pmatrix} \tag{14}$$

in case of flyby one. Let us assume, that $\mathbf{f}(T,\ 0,\ \mathbf{p}_{\mathrm{vo}}(0),\ \dot{\mathbf{p}}_{\mathrm{vo}}(0)) = \mathbf{b}$, where vectors $\mathbf{p}_{\mathrm{vo}}(\tau)$, $\dot{\mathbf{p}}_{\mathrm{vo}}(\tau)$ are considered as function of the continuation parameter τ. The main equation of the continuation method has follows form in this case:

$$\frac{d\mathbf{z}}{d\tau} = -\mathbf{f}_{\mathbf{z}}^{-1}(\mathbf{z})\left(\mathbf{b} + \frac{\partial \mathbf{f}}{\partial \tau}\right),\ \ \mathbf{z}(0) = \mathbf{z}_0, \tag{15}$$

where $\mathbf{z}(\tau) = \left(\mathbf{p}_{\mathrm{vo}}, \dot{\mathbf{p}}_{\mathrm{vo}}\right)^{\mathrm{T}}$.

It is possible to use difference derivatives to calculate matrix $\mathbf{f}_{\mathbf{z}}$ and vector $\mathbf{f}_\tau$ in the (15). But accuracy of difference derivatives does not satisfactory one frequently. So, it is preferred to calculate $\mathbf{f}_{\mathbf{z}}$ and $\mathbf{f}_\tau$ by means of associated integration of the (9) and variations of the state vector and adjoint variables. The complete system of the differential equations is follows:

$$
\left.\begin{aligned}
&\frac{\mathrm{d}^2\mathbf{x}}{\mathrm{d}t^2}=\Omega_{\mathbf{x}}+\varphi_1(\tau)\mathbf{p}_{\mathrm{v}}+\varphi_2(\tau)\frac{a\varepsilon}{p_{\mathrm{v}}}\mathbf{p}_{\mathrm{v}},\\
&\frac{\mathrm{d}^2\mathbf{p}_{\mathrm{v}}}{\mathrm{d}t^2}=\Omega_{\mathbf{xx}}\mathbf{p}_{\mathrm{v}},\\
&\frac{\mathrm{d}^2}{\mathrm{d}t^2}\left(\frac{\partial\mathbf{x}}{\partial\mathbf{p}_{\mathrm{vo}}}\right)=\Omega_{\mathbf{xx}}\frac{\partial\mathbf{x}}{\partial\mathbf{p}_{\mathrm{vo}}}+\varphi_1(\tau)\frac{\partial\mathbf{p}_{\mathrm{v}}}{\partial\mathbf{p}_{\mathrm{vo}}}+\frac{a\varepsilon}{p_{\mathrm{v}}}\varphi_2(\tau)\left[\mathbf{E}-\frac{1}{p_{\mathrm{v}}^2}\mathbf{p}_{\mathrm{v}}\mathbf{p}_{\mathrm{v}}^{\mathrm{T}}\right]\frac{\partial\mathbf{p}_{\mathrm{v}}}{\partial\mathbf{p}_{\mathrm{vo}}},\\
&\frac{\mathrm{d}^2}{\mathrm{d}t^2}\left(\frac{\partial\mathbf{p}_{\mathrm{v}}}{\partial\mathbf{p}_{\mathrm{vo}}}\right)=\frac{\partial}{\partial\mathbf{x}}\left(\Omega_{\mathbf{xx}}\mathbf{p}_{\mathrm{v}}\right)\frac{\partial\mathbf{x}}{\partial\mathbf{p}_{\mathrm{vo}}}+\Omega_{\mathbf{xx}}\frac{\partial\mathbf{p}_{\mathrm{v}}}{\partial\mathbf{p}_{\mathrm{vo}}},\\
&\frac{\mathrm{d}^2}{\mathrm{d}t^2}\left(\frac{\partial\mathbf{x}}{\partial\tau}\right)=\Omega_{\mathbf{xx}}\frac{\partial\mathbf{x}}{\partial\tau}+\varphi_1(\tau)\frac{\partial\mathbf{p}_{\mathrm{v}}}{\partial\tau}+\frac{a\varepsilon}{p_{\mathrm{v}}}\varphi_2(\tau)\left[\mathbf{E}-\frac{1}{p_{\mathrm{v}}^2}\mathbf{p}_{\mathrm{v}}\mathbf{p}_{\mathrm{v}}^{\mathrm{T}}\right]\frac{\partial\mathbf{p}_{\mathrm{v}}}{\partial\tau}+\\
&\qquad+\frac{\partial\varphi_1}{\partial\tau}\mathbf{p}_{\mathrm{v}}+\frac{\partial\varphi_2}{\partial\tau}\frac{a\varepsilon}{p_{\mathrm{v}}}\mathbf{p}_{\mathrm{v}},\\
&\frac{\mathrm{d}^2}{\mathrm{d}t^2}\left(\frac{\partial\mathbf{p}_{\mathrm{v}}}{\partial\tau}\right)=\frac{\partial}{\partial\mathbf{x}}\left(\Omega_{\mathbf{xx}}\mathbf{p}_{\mathrm{v}}\right)\frac{\partial\mathbf{x}}{\partial\tau}+\Omega_{\mathbf{xx}}\frac{\partial\mathbf{p}_{\mathrm{v}}}{\partial\tau},
\end{aligned}\right\}\tag{16}
$$

where **E** is identity matrix. The third equation in the (16) must include terms with partial derivative of the ε with respect to $\mathbf{p}_{\mathrm{vo}}$, and the fifth one must include partial derivative of the ε with respect to τ. These derivatives has multiplier $\delta(p_{\mathrm{v}}-1)$ (delta-function $\delta(p_{\mathrm{v}}-1)=\infty$ if $p_{\mathrm{v}}=1$ and it is equal to 0 otherwise). We will take into account these terms if we would interrupt integration of the (16) when $p_{\mathrm{v}}=1$ to change magnitude of the $\mathrm{d}(\partial\mathbf{x}/\partial\mathbf{p}_{\mathrm{vo}})/\mathrm{d}t$, $\mathrm{d}(\partial\mathbf{x}/\partial\tau)/\mathrm{d}t$ in every of these inner points. Indeed, these equation can be written in the form

$$
\frac{d^2}{dt^2}\left(\frac{\partial\mathbf{x}}{\partial e}\right)=\frac{d}{dt}\left(\frac{\partial\mathbf{v}}{\partial e}\right)=\mathbf{f}_1+\delta\frac{s}{p_{\mathrm{v}}}\mathbf{f}_2,\tag{17}
$$

where e is τ or one of the components of the vector $\mathbf{p}_{\mathrm{vo}}$, $\mathbf{f}_1$ and $\mathbf{f}_2$ are continuous functions and $s=\mathrm{sign}(\dot{p}_{\mathrm{v}})\ \mathbf{p}_{\mathrm{v}}^{\mathrm{T}}(d\mathbf{p}_{\mathrm{v}}/de)$ (the second term with δ-multiplier is absence in the (16); $\mathbf{f}_2$ is follows: $\mathbf{f}_2=a\varphi_2(\tau)\mathbf{p}_{\mathrm{v}}/p_{\mathrm{v}}$). Let us suppose that $p_{\mathrm{v}}(t_1)=1$. From equation (17) follows

$$
\left.\frac{\partial\mathbf{v}}{\partial e}\right|_{\mathrm{t=t^+}}=\left.\frac{\partial\mathbf{v}}{\partial e}\right|_{\mathrm{t=t^-}}+\lim_{\Delta t\to 0}\left[\int_{\mathrm{t_1-\Delta t}}^{\mathrm{t_1+\Delta t}}\mathbf{f}_1dt+\int_{\mathrm{t_1-\Delta t}}^{\mathrm{t_1+\Delta t}}\delta\frac{s}{p_{\mathrm{v}}}\mathbf{f}_2dt\right].\tag{18}
$$

The limit of the first integral is equal to 0. It is necessary to change of variable t by p_{v} to calculate the second integral: $dt=(1/s_{\mathrm{t}})dp_{\mathrm{v}}$, where $s_{\mathrm{t}}=\frac{1}{p_{\mathrm{v}}}\mathbf{p}_{\mathrm{v}}^{\mathrm{T}}\dot{\mathbf{p}}_{\mathrm{v}}$. Resultant expression is

$$
\left.\frac{\partial\mathbf{v}}{\partial e}\right|_{\mathrm{t=t^+}}=\left.\frac{\partial\mathbf{v}}{\partial e}\right|_{\mathrm{t=t^-}}+\frac{s}{s_{\mathrm{t}}p_{\mathrm{v}}}\mathbf{f}_2.\tag{19}
$$

Equation (19) should be used in the all inner points where $p_{\mathrm{v}}=1$.

Initial conditions (6) should be expanded by follows expressions:

$$\frac{\partial \mathbf{x}(0)}{\partial \mathbf{p}_{vo}} = \frac{\partial \mathbf{x}(0)}{\partial \mathbf{p}_{vo}} = \frac{\partial \mathbf{p}_v(0)}{\partial \dot{\mathbf{p}}_{vo}} = \frac{d}{dt}\left(\frac{\partial \mathbf{p}_v(0)}{\partial \mathbf{p}_{vo}}\right) = 0,$$

$$\frac{d}{dt}\left(\frac{\partial \mathbf{x}(0)}{\partial \mathbf{p}_{vo}}\right) = \frac{V_\infty}{p_{vo}}\left(\mathbf{E} - \frac{1}{p_{vo}^2}\mathbf{p}_{vo}\mathbf{p}_{vo}^{\mathrm{T}}\right), \quad \frac{\partial \mathbf{p}_v(0)}{\partial \mathbf{p}_{vo}} = \frac{d}{dt}\left(\frac{\partial \mathbf{p}_v(0)}{\partial \dot{\mathbf{p}}_{vo}}\right) = \mathbf{E}, \tag{20}$$

Let us assume that solution of the LP-problem (10) is known: $\mathbf{p}_v(0) = \mathbf{p}_{vo}^{LP}$(one efficient numerical method to solve LP problem see in another our paper[2]). Let us suppose $\mathbf{p}_{vo}(0)=\mathbf{p}_{vo}^{LP}$ Vector **b** is equal to 0 in this case. Equation (9) remains invariant with respect to constant parameter k, if we norm adjoint vector

$$\mathbf{p}_v = \mathbf{p}_v^{LP}/(k\, p_{vo}^{LP}), \tag{21}$$

and if it is assumed

$$\varphi_1(\tau) = k\, \mathbf{p}_{vo}^{LP} \cdot (1 - \tau),\ \varphi_2(\tau) = \tau. \tag{22}$$

Equations (21) and (22) have following meaning. Norm of the $\mathbf{p}_{vo}^{LP}$ is arbitrary. It is essentially less than 1 in case of the low thrust problem. This fact means that $\varepsilon \equiv 0$ in the beginning of the continuation process if $\mathbf{p}_v(0) = \mathbf{p}_{vo}^{LP}$. The continuation process in this initial interval of τ degenerates into increase of the p_v. This interval is finished at the τ_1: $p_{v\max}(\tau_1) = \max\limits_t p_v(t;\tau_1) = 1$. The result of the continuation process in this interval is equivalent to the normalization

$$\mathbf{p}_v = \mathbf{p}_v^{LP}/p_{v\max}(\tau_1). \tag{23}$$

It is necessary to find τ_1 and $p_{v\max}(\tau_1)$ to normalize $\mathbf{p}_v$ in according with (14). It is difficult enough problem and it is not necessary from the point of view construction of the numerical continuation process. Moreover, easier normalization (21) has some advantage in comparison with (23). Use either (23) or (21) allows to eliminate initial unproductive interval of continuation, but normalization (21) allows also to choose number of the burn arcs at the $\tau = 0$. This possibility is demonstrated in the Fig.3. Here it is shown curve of dependency of the p_v with respect to time in case of LP problem. There are two burn arcs (from t_3 to t_4 and from t_5 to T) at the beginning of the continuation process if we choose $k=k_o=1$. The coast arc precedes these burn arcs. It is unique burn arc if $k=k_1$ (from t_6 to t_7). Two burn arcs (from 0 to t_1 and from t_2 to T) exist if we choose $k_2<1$. The initial coast arc is absent in this case.

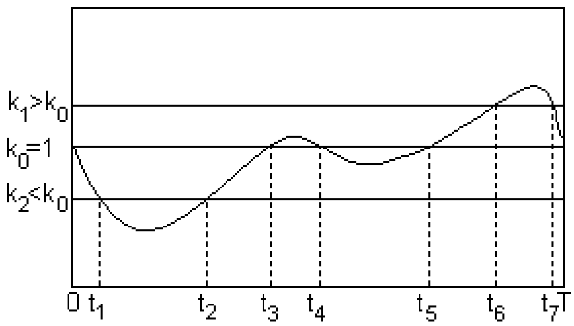

Fig.3. Use of parameter k to choose number of the burn arcs.

Numerical examples

There are presented three examples of the numerical solutions of the LP problem and their continuation into the CEV solutions. The first example is transfer between two elliptical orbits, the second one is transfer from low Earth orbit to geosynchronous orbit, and the third one is 45-degree phase change of the orbital motion.

Table 2

The examples of the numerical solutions

Parameter		Problem 1	Problem 2	Problem 3
Start	Orbit	$p_o=1$, $e_o=0.1$, $\omega_o=\pi$	$p_o=1$, $e_o=0$	$p_o=1$, $e_o=0$
	r	(1.1037, -0.1212, 0)	(1,0,0)	(1,0,0)
	v	(0.1092, 0.8940, 0)	(0,1,0)	(0,1,0)
Finish	Orbit	$p_f=1$, $e_f=0.4$, $\omega_f=0$	$p_f=6.238$, $e_f=0$	$p_f=1$, $e_f=0$
	r	(0.4963, 0.6295, 0)	(-3.4882, -5.1715, 0)	(0.8763, 0.4818, 0)
	v	(-0.7853, 1.0191, 0)	(0.3316, -0.2237, 0)	(-0.4818, 0.8763, 0)
T		7.5	13.7	6.0
LP model				
$\mathbf{p}_{vo}$		(0.003950, 0.054272, 0)	(-0.050218, 0.069321, 0)	(-0.021830, -0.056861, 0)
$d\mathbf{p}_{vo}/dt$		(-0.024306, 0.005009, 0)	(-0.048960, 0.042200, 0)	(0.055215, 0.017111, 0)
CEV model				
a		0.05	0.10	0.05
$\mathbf{p}_{vo}$		(0.093948, 1.820225, 0)	(-2.104856, 9.328105, 0)	(-0.829581, -2.217508, 0)
$d\mathbf{p}_{vo}/dt$		(-0.807759, 0.194261, 0)	(-7.727214, 2.106728, 0)	(2.165689, 0.641663, 0)
Time of switching	1	1.659687	6.085439	1.058080
	2	2.604417	7.646246	4.941918
	3	5.121851	-	-
	4	5.812537	-	-
k		0.5	1.0	0.9

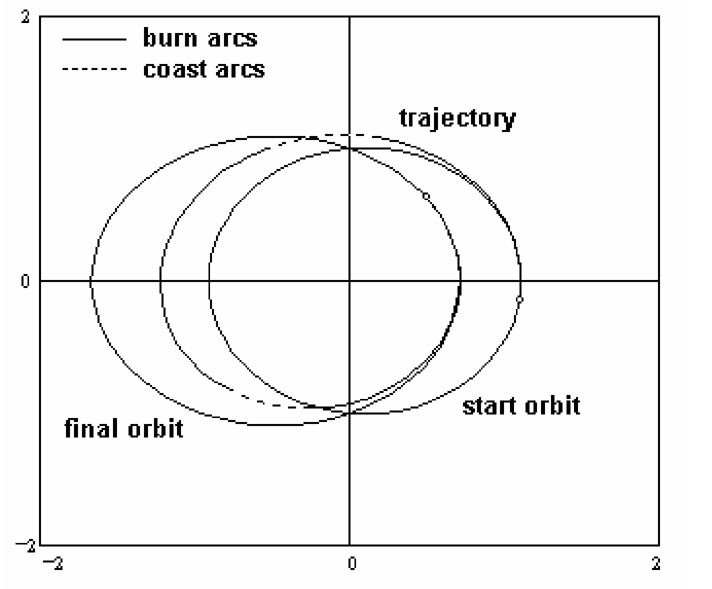

Fig.4. Optimal CEV trajectory (first example in the table 2)

Let us consider the first example more detailed. Trajectory of motion is presented in the Fig.4. This trajectory includes three burn arcs. Fig.5 presents magnitude of the optimal thrust acceleration in cases of LP and CEV models. Dashed line corresponds to k=0.5. This magnitude of k provides existence of the three burn arcs at the beginning of the continuation process. Fig.6 shows angle between thrust and x-axis both in case of LP model and CEV one. The difference between these angles is quite small. This example demonstrates that LP solution is a good approximation of the CEV one.

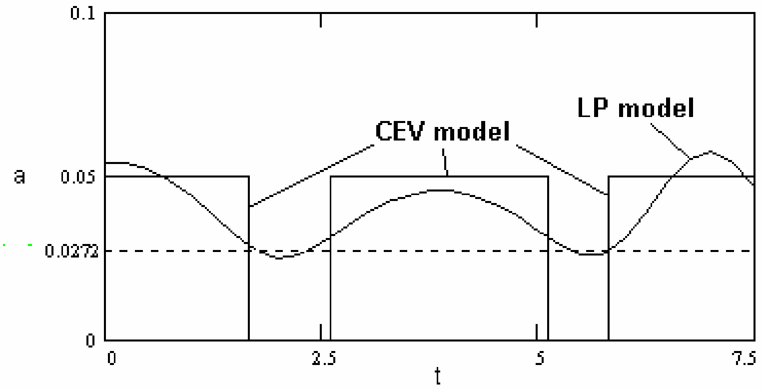

Fig.5. Thrust acceleration (first example in the table 2)

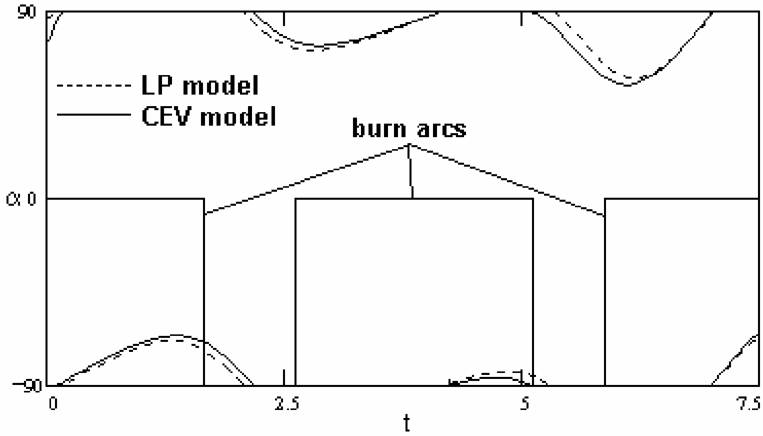

Fig.6. Angle between thrust and x-axis (first example in the table 2)

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